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FRACTAL SCALING IN CELLULAR AUTOMATA SIMULATIONS OF DISSIPATIVE DYNAMICAL SYSTEMS

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13. ABSTRACT (Maximum 200 words)

A thorough understanding of simple processes often provides insight into the nature of complex systems. In this investigation, cellular automata are used to study the natural evolution of dissipative dynamical systems and a new technique for extracting fractal scaling parameters is described. The cellular automata model the evolution of a complex structure with the properties of a self-organized critical (SOC) system as suggested by Bak, Tang, and Wiesenfeld. We have demonstrated that the evolving structures exhibit fractal scaling and that the fractal measures vary in response to changes in rules governing the dynamics of the system. We also demonstrate that the range over which fractal scaling occurs increases smoothly as the model evolves and that subcritical, as well as critical, avalanche distributions have the form predicted for a size-effect limited SOC system. The subcritical scaling is fractal over limited ranges of scales and the scaling range correlates with the size of subdomain structures.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

INTRODUCTION

Bak, Tang, and Wiesenfeld (ref 1) introduced the notion of self-organized critical (SOC) systems to provide a consistent explanation for the fractal spatial structures, power law distributions, and flicker noise commonly observed in spatially-extended driven dissipative dynamical systems and suggested that sand piles were examples of such systems. They proposed that the process of adding grains of sand at random locations within a grid produces a sand pile whose geometric configuration evolves toward a critical form. The critical state is an attractor for the dynamics of the system. The implications of the SOC concept for the distributions of avalanches in "sand pile numerical models" have been extensively explored (refs 1-4). It was demonstrated that avalanche sizes in finite sand pile models had the form of "cutoff hyperbolic distributions" consistent with the predictions of Bak, Tang, and Wiesenfeld (ref 1). These models demonstrate how complex fractal surfaces evolve from simple system dynamics. The state of the sand pile at any time is given by the height of the pile $h(x,y)$ at points $\{x,y\}$ on a two-dimensional lattice. The automata are governed by rules that define the conditions for site stability and the nature of the local rearrangements that occur when a site becomes unstable.

Kadanoff et al. (ref 2) showed that the distribution of avalanche sizes that occurs when an SOC state is perturbed can be fit over extensive ranges by power laws. Jensen et al. (ref 3) and Meisel and Cote (ref 5) demonstrate that cutoff power law distributions of avalanche sizes give rise to flicker noise in the power spectral density. Thus, time-scale invariance (flicker noise) over increasing time scales and length-scale invariance (fractal scaling) over increasing spatial scales develop as the model is "driven" through a sequence of subcritical self-organized states. The range of fractal scaling correlates with the size-effect parameters in the cutoff power law avalanche size distributions and in the critical state, fractal scaling may be observed over essentially all scales.

However, a quantitative analysis of the geometric scaling properties of the evolving automata structures has not previously been reported. The limited point sets generated in typical sand pile simulations are insufficient for the application of standard fractal analysis techniques. Therefore, a new procedure for extracting fractal parameters has been developed (ref 6). This new technique is inherently parallel and its fast convergence makes it well suited to the study of limited fractal subsets. The algorithm has been implemented on a multicomputer environment using a subset of TROLLIUS (Ohio State University and Cornell Research Foundation) called LAM (ref 7). LAM is a programming environment and development system for a message passing multicomputer comprised entirely of UNIX workstations.

THE SAND PILE DYNAMICAL MODELS

The cellular automata employed in this study are based on a two-dimensional regular lattice of cells using the rules similar to those given for model 2 of Reference 2. The basic variable is $z(i,j)$ where (i,j) are the spatial indices and z represents the height of the lattice. The system is initialized to a planar surface, $z(i,j) = 0$. Particles are added at random locations $z(i,j)$ such that $z(i,j) \rightarrow z(i,j) + 1$ as long as all the sites are stable. After a particle is added and after particle rearrangements, the stability of all lattice sites is determined.

The stability of the site $\{i,j\}$ is determined by $z(i,j)$ and the nearest neighbor z values. In the model treated here, a site becomes unstable when

$$\sum_{k=-1}^{k=1}\{(z(i,j)-z(i,j-k))H(z(i,j)-z(i,j-k))+(z(i,j)-z(i-k,j))H(z(i,j)-z(i-k,j))\} > \sigma_C$$

where H is the Heaviside function and σ_C is the stability parameter.

If site $\{i,j\}$ becomes unstable, $z(i,j)$ is reduced by

$$\sum_{k=-1}^{k=1}\{H(z(i,j)-z(i,j-k))+H(z(i,j)-z(i-k,j))\}$$

and the height of the corresponding nearest neighbor cells is incremented by 1 if and only if their z-value is less than $z(i,j)$. Note that in Reference 3, the stability of a site is determined by all nearest neighbors (so that a higher z neighbor site tends to stabilize a given site), and that particles are allowed to slide "uphill." If all sites are stable, a new particle is added to the system.

The boundaries are treated as cells with $z(x,y) = 0$. The cellular automata rule is applied recursively to the cells whose state is affected by the unstable site, and the diffusion process continues until there are no more unstable sites. No new particles are added to the lattice until the lattice stabilizes. The number of sites that is changed as the lattice reorganizes after a particle is added is the size of the resulting avalanche, and the total number of particles that has been added to the system is denoted N.

D BY TRIANGULATION

The dynamical model under investigation yields a single-valued sand pile surface $z(i, j)$. It is, therefore, possible to define a hierarchy of approximations of the surface in terms of triangles of different sizes in a simple unambiguous way. The extent of the lattice models studied has been adjusted to facilitate the triangular approximations. Taking

$$L = N = 2^n + 1 \text{ for integer } n$$

enables one to define sets of triangles whose vertices coincide with subsets of points on the lattice and whose projections in the xy-plane are right triangles of side lengths Y, which we refer to as "yardsticks," given by

$$Y(m) = 2^m \text{ for } m = \{0,1,...n\}$$

The small $Y(m)$ approximations to fractal surfaces scale approximately according to

$$\frac{d\ln(A(m))}{d\ln(Y(m))} \xrightarrow[m\to 0]{} 2 - D$$

where the area $A(m)$ is the surface area for the triangulation based on $Y(m)$ by $Y(m)$ cells, and D is a fractal dimension. $A(m)$ can be expressed in terms of individual triangle areas

$$A(m) \equiv \sum A_i\ (m)$$

Note that the smallest yardstick in the set is $Y(0) = 1$, which is as far as one can go in the analysis of a surface defined on a discrete lattice of points.

A detailed description of the algorithm has been published (ref 6). The reliability of the algorithm has been successfully tested with Euclidean and "Brown" surfaces (ref 8). Brown constructions were selected for the tests because elevations are single-valued, as required by the algorithm, and because D can be adjusted to have any value between 2 and 3.

LAM IMPLEMENTATION

The triangulation algorithm is capable of reliably determining fractal scaling parameters of relatively sparse, nonhomogeneous data sets. However, the generation of local D-maps is numerically expensive. Therefore, a suite of homogeneous parallel procedures, which exploits the inherent parallelism of the computational problem, has been developed to map the triangulation algorithm to a multicomputer platform using a subset of TROLLIUS (product of Ohio State University and the Cornell Research Foundation) called LAM (ref 7) for interprocess communication. The multicomputer is restricted to a network of machines that run LAM as a native application under UNIX. LAM generally supports only general purpose UNIX machines connected via a local area network or the internet. LAM is a node-oriented computing environment that uses a unique identifier assigned to each node (nodeid) as the primary synchronization for communication. The nodes are usually fully connected (maximum 1-hop distance) since the network is generally a shared resource. In our parallel implementation of the triangulation algorithm, a master process assigns equal-sized regions of the structure to slave processing nodes for computation of local D-map subsets. The processing nodes return the results to the master process, where the D-map is assembled and the slave processes are assigned new regions to analyze. Although this approach does not exploit the communication advantages of a fully connected network, the load balancing resulted in a performance improvement directly proportional to the number of workstations. This is likely due to the high computation/communication ratio and the relatively small (15) number of workstations making up the multicomputer.

RESULTS

Results were obtained for structures with $\sigma_c = 20$ through 90 in increments of 10 on 513 x 513 lattices for extensive ranges of N. We refer to the configuration obtained for $\sigma_c = 20$ and $N = 7 \times 10^8$ particles as the $\sigma_c = 20$ structure, and that for $\sigma_c = 40$ and $N = 7 \times 10^8$ as the $\sigma_c = 40$ structure.

Figure 1 shows interpolated shading representations of central 40 x 40 regions of the $\sigma_c = 20$ and the $\sigma_c = 40$ structures.

Figure 2 plots D versus σ_c for central regions of "close-to-critical" sand pile structures. As σ_c increases, larger N values are required to reach critical sand pile configurations. However, if N is chosen too large, boundary effects "diffuse" into the region being analyzed. Thus, the irregularities in the D versus σ_c results are consequences of varying degrees of convergence toward critical configurations rather than characteristics of the model.

Figure 3 is a typical central cross section of the $\sigma_c = 20$ structure showing the boundary-effected regions and the relative variations in z. The boundary-effected region has diffused into the central 257 x 257 square; however, the central 129 x 129 square has not yet been impinged upon by boundary effects.

Figure 4 shows distributions of local D values (obtained by analysis of the corresponding D-maps) for the $\sigma_c = 20$ and 40 structures. Boundary effects are represented by the local D values near 2.0 in the central 257 x 257 region for the $\sigma_c = 20$ structure. These are consequences of the fact that the steep "walls" seen in Figure 3 are nearly flat. The effect of varying σ_c for regions free of boundary influences is apparent in the results presented for the central 129 x 129 regions: Generally, as σ_c increases, the mean value of D and the breadth of the distribution of local D values increase.

Figure 5 shows how the structure with $\sigma_c = 90$ passes through subcritical regions as more particles are added. The useful scaling range in the least squares fit (the range over which fractal scaling is obtained) increases from 3 doublings at $N = 10^7$ to 6 doublings at $N = 10^9$. The range of fractal scaling in the subcritical regions correlates with the magnitude of the cutoff in the size-effect limited hyperbolic distribution of avalanche sizes.

SUMMARY

The simulations demonstrate that complex SOC systems can arise from the simple local dynamics of cellular automata. In the present case, a scale-invariant structure with correlating size-effect limited hyperbolic distributions of avalanches evolves from a planar surface in a system governed by simple uniform local dynamical rules. The diffusion process leads to uniform scaling well beyond the nearest neighbor distances that are directly involved when the system is perturbed.

A new analytic technique, based on triangular tessellations of single-valued surfaces in 3-space, has been applied to extract the fractal scaling parameters from the relatively sparse data sets generated by the present simulations. Since the algorithm is numerically expensive to apply to the determination of local D-maps of heterogeneous surfaces, the computational problem has been mapped to a multicomputer platform comprised of UNIX workstations using a subset of TROLLIUS called LAM for interprocess communication. Computation of large sets of local D-maps is presently impractical without the use of a highly dedicated computer ensemble.

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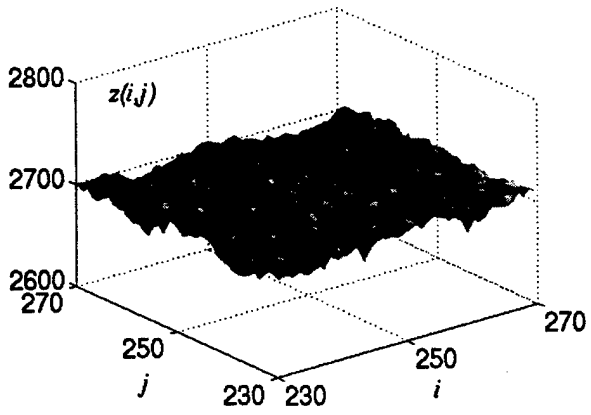

Figure 1a. Interpolated Shading Representation of Central Region of Sand Pile Structure for $\sigma_c = 20$ and $N = 7x10^8$.

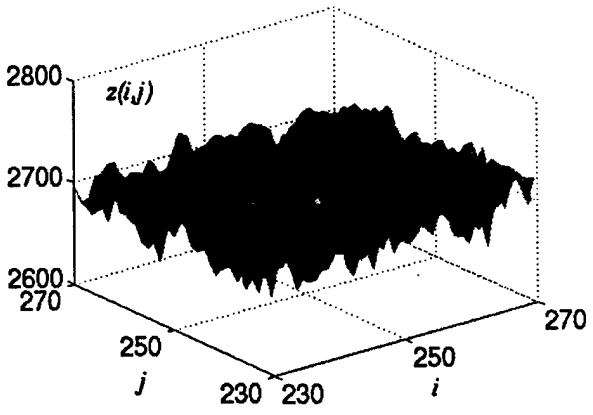

Figure 1b. Interpolated Shading Representation of Central Region of Sand Pile Structure for $\sigma_c = 40$ and $N = 7x10^8$.

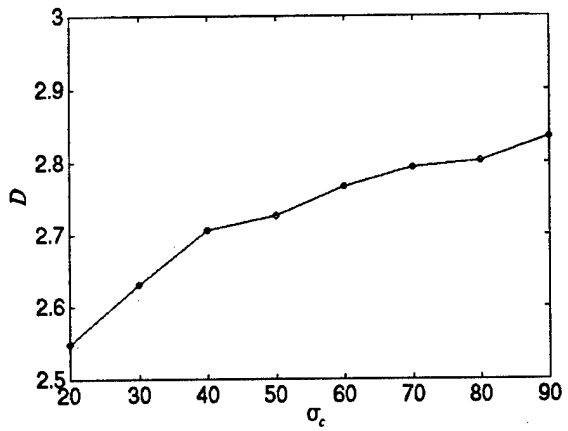

Figure 2. Fractal Dimension D vs. σ_c.

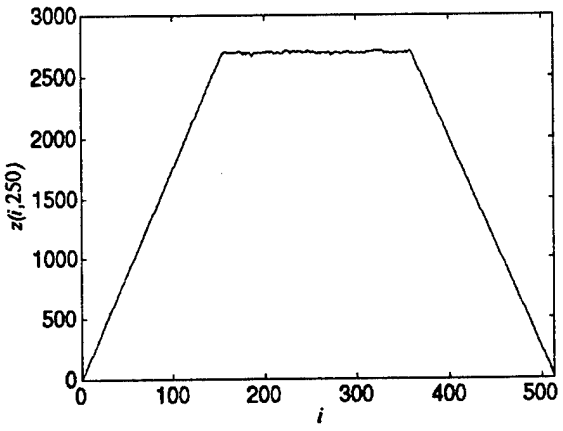

Figure 3. Cross Section of Sand Pile Structure for $\sigma_c = 20$ and $N = 7x10^8$.

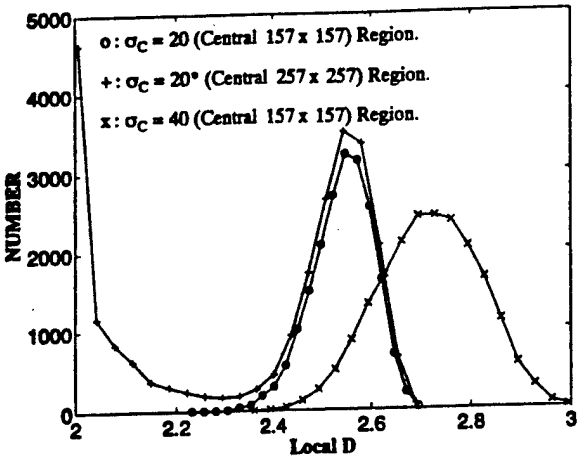

Figure 4. Distributions of Local D Values.

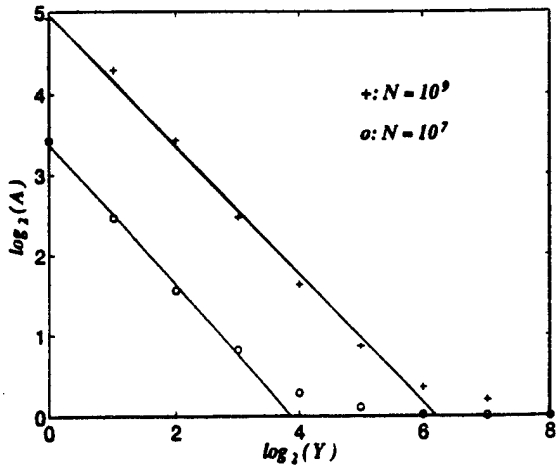

Figure 5. Area vs. Yardstick plots for σ_C = 90. The lines are least squares fit to the first 4 points for the N= 10^7 structure and to the first 7 points for the N= 10^9 structure.

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